

The differential equation method: evaluation of complicated Feynman integrals

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Abstract.

We discuss a progress in calculation of Feynman integrals which has been done with help of the differential equation method and demonstrate the results for a class of two-point two-loop diagrams.

The idea of the differential equation method (DEM) (see [1,2]): to apply the integration by parts procedure [3] to an internal n -point subgraph of a complicated Feynman diagram and later to represent new complicated diagrams, obtained here, as derivatives in respect of corresponding mass of the initial diagram.

Thus, we have got the differential equations for the initial diagram. The inhomogeneous terms contain only more simpler diagrams. These simpler diagrams have more trivial topological structure and/or less number of loops [1] and/or ends [2].

Applying the procedure several times, we will able to represent complicated Feynman integrals (FI) and their derivatives (in respect of internal masses) through a set of quite simple well-known diagrams. Then, the results for the complicated FI can be obtained by integration several times of the known results for corresponding simple diagrams.

Sometimes it is useful (see [4]) to use external momenta (or some their functions) but not masses as parameters of integration.

The recent progress in calculation of Feynman integrals with help of the DEM.

1. The articles [5] and [6]:

a) The set of two-point two-loop FI with one- and two-mass thresholds (see Fig.1) has been evaluated by a combination of DEM and Veretin programs for calculation of first terms of FI small-moment expansion. The results are given below (some of them have been known before (see discussions in [5])).

b) The set of three-point two-loop FI with one- and two-mass thresholds has been evaluated (the results of

some of them has been known before (see [6])) by a combination of DEM and Veretin programs for calculation of first terms of FI small-moment expansion. The finite parts of the integrals can be written in terms of the generalized Nielsen polylogarithms (see [8]).

2. The article [7]:

The full set of two-loop onshell master diagrams has been evaluated by DEM and Kalmykov programs for calculation of first terms of FI small-moment expansion. It has been observed that finite parts of all such integrals without subdivergences can be written in terms of three constants, two for the real part, ζ_3 and $\pi Cl_2(\pi/3)$, and one for the imaginary part, $\pi\zeta_2$.

3. The articles [9]:

The set of three-point and four-point two-loop massless FI has been evaluated.

Here we demonstrate the results of FI are displayed on Fig.1.

We introduce the notation for polylogarithmic functions:

$$\begin{aligned} \text{Li}_a(z) &= S_{a-1,1}(z), \\ S_{a+1,b}(z) &= \frac{(-1)^{a+b}}{a!b!} \int_0^1 \frac{\log^a(t) \log^b(1-zt)}{t} dt. \end{aligned}$$

We introduce also the following two variables

$$z = \frac{q^2}{m^2}, \quad y = \frac{1 - \sqrt{z/(z-4)}}{1 + \sqrt{z/(z-4)}}.$$

Then

$$\begin{aligned} q^2 \cdot I_1 &= -\frac{1}{2} \log^2(-z) \log(1-z) - 2 \log(-z) \text{Li}_2(z) \\ &\quad + 3 \text{Li}_3(z) - 6 S_{1,2}(z) - \log(1-z) [\zeta_2 + 2 \text{Li}_2(z)], \\ q^2 \cdot I_5 &= 2 \zeta_2 \log(1+z) + 2 \log(-z) \text{Li}_2(-z) \\ &\quad + \log^2(-z) \log(1+z) + 4 \log(1+z) \text{Li}_2(z) - 2 \text{Li}_3(-z) \end{aligned}$$

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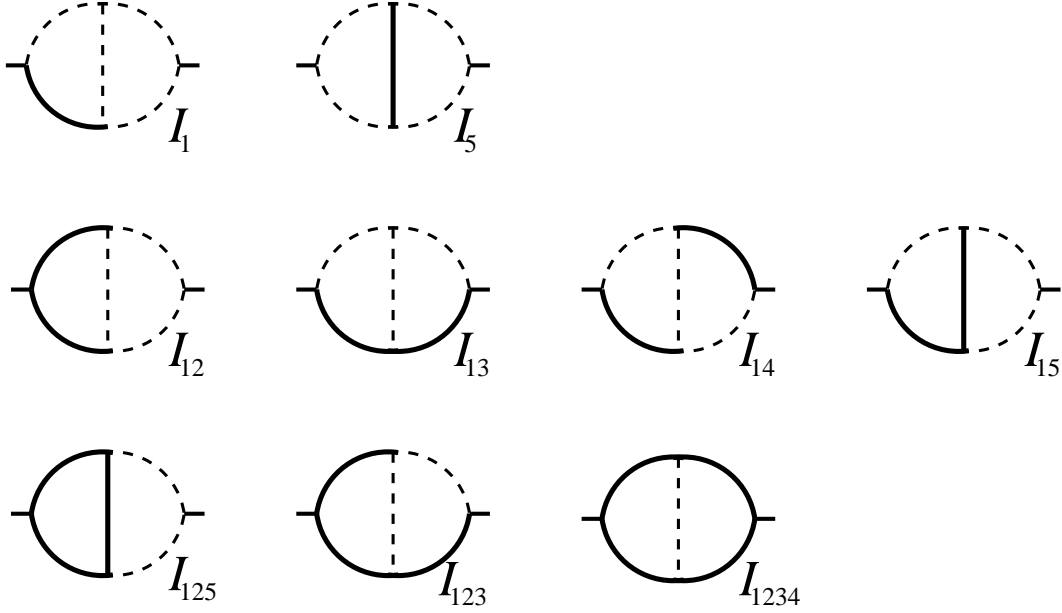


FIGURE 1. Two-loop selfenergy diagrams. Solid lines denote propagators with the mass m ; dashed lines denote massless propagators.

$$\begin{aligned}
& -2\text{Li}_3(z) + 2S_{1,2}(z^2) - 4S_{1,2}(z) - 4S_{1,2}(-z), \\
q^2 \cdot I_{12} &= \text{Li}_3(z) - 6\zeta_3 - \zeta_2 \log y - \frac{1}{6} \log^3 y - 4 \log y \text{Li}_2(y) \\
& + 4\text{Li}_3(y) - 3\text{Li}_3(-y) + \frac{1}{3} \text{Li}_3(-y^3), \\
q^2 \cdot I_{13} &= -6S_{1,2}(z) - 2 \log(1-z) [\zeta_2 + \text{Li}_2(z)], \\
q^2 \cdot I_{14} &= \log(2-z) [\log^2(1-z) - 2 \log(-z) \log(1-z) \\
& - 2\text{Li}_2(z)] - \frac{2}{3} \log^3(1-z) + \log(-z) \log^2(1-z) \\
& - 2\zeta_2 \log(1-z) - S_{1,2}(1/(1-z)^2) + 2S_{1,2}(1/(1-z)) \\
& + 2S_{1,2}(-1/(1-z)) + \frac{1}{3} \log^3 y + \log^2 y [2 \log(1+y^2) \\
& - 3 \log(1-y+y^2)] - 6\zeta_3 - \text{Li}_3(-y^2) + \frac{2}{3} \text{Li}_3(-y^3) \\
& - 6\text{Li}_3(-y) + 2 \log y [\text{Li}_2(-y^2) - \text{Li}_2(-y^3) + 3\text{Li}_2(-y)], \\
q^2 \cdot I_{15} &= 2\text{Li}_3(z) - \log(-z) \text{Li}_2(z) + \zeta_2 \log(1-z) \\
& + \frac{1}{6} \log^3 y - \frac{1}{2} \log^2 y [8 \log(1-y) - 3 \log(1-y+y^2)] \\
& - 6\zeta_3 - \frac{1}{3} \text{Li}_3(-y^3) + 3\text{Li}_3(-y) + 8\text{Li}_3(y) \\
& + \log y [\text{Li}_2(-y^3) - 3\text{Li}_2(-y) - 8\text{Li}_2(y)], \\
q^2 \cdot I_{123} &= -\zeta_2 [\log(1-z) + \log y] - 6\zeta_3 \\
& - \frac{3}{2} \log(1-y+y^2) \log^2 y \\
& + \text{Li}_3(-y^3) - 9\text{Li}_3(-y) - 2 \log y [\text{Li}_2(-y^3) - 3\text{Li}_2(-y)], \\
q^2 \cdot I_{125} &= -2 \log^2 y \log(1-y) - 6\zeta_3 + 6\text{Li}_3(y) \\
& - 6 \log y \text{Li}_2(y),
\end{aligned}$$

$$\begin{aligned}
q^2 \cdot I_{1234} &= -6\zeta_3 - 12\text{Li}_3(y) - 24\text{Li}_3(-y) + 8 \log y [\text{Li}_2(y) \\
& + 2\text{Li}_2(-y)] + 2 \log^2 y [\log(1-y) + 2 \log(1+y)].
\end{aligned}$$

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